Zeros Of Bernoulli, Generalized Bernoulli, And Euler Polynomials

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Research Article

Dynamics of the Zeros of Analytic Continued (h, q)-Euler Polynomials

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In this paper, we study that the (h, q)-Euler numbers $E_{n,q}^{(h)}$ and (h, q)-Euler polynomials $E_{n,q}^{(h)}(x)$ are analytic continued to $E_q^{(h)}(s)$ and $E_q^{(h)}(s, w)$. We investigate the new concept of dynamics of the zeros of analytic continued polynomials related to solution of Bernoulli equation. Finally, we observe an interesting phenomenon of "scattering" of the zeros of $E_q^{(h)}(s, w)$.

1. Introduction

By using software, many mathematicians can explore concepts much more easily than in the past. The ability to create and manipulate figures on the computer screen enables mathematicians to quickly visualize and produce many problems, examine properties of the figures, look for patterns, and make conjectures. This capability is especially exciting because these steps are essential for most mathematicians to truly understand even basic concept. Recently, the computing environment would make more and more rapid progress and there has been increasing interest in solving mathematical problems with the aid of computers. Mathematicians have studied different kinds of the Euler, Bernoulli, Tangent, and Genocchi numbers and polynomials. Numerical experiments of Bernoulli polynomials, Euler polynomials, Genocchi polynomials, and Tangent polynomials have been the subject of extensive study in recent year and much progress has been made both mathematically and computationally (see [1–18]). Throughout this paper, we always make use of the following notations: $\mathbb{N}$ denotes the set of natural numbers, $\mathbb{N}_0$ denotes the set of nonnegative integers, $\mathbb{Z}$ denotes the set of integers, $\mathbb{R}$ denotes the set of real numbers, and $\mathbb{C}$ denotes the set of complex numbers. Let q be a complex number with $|q| < 1$ and $h \in \mathbb{Z}$. Bernoulli equation is one of the well known nonlinear differential equations of the first order. It is written as

$$\frac{dy}{dt} + p(x)\,y = g(x)\,y^m \quad (m \text{ any real number}), \tag{1}$$

where $p(x)$ and $g(x)$ are continuous functions. For $m = 0$ and $m = 1$ the equation is linear, and otherwise it is nonlinear. When $m = 2$, the Bernoulli equation has the solution which is the function of exponential generating function of the Euler numbers. Simsek [18] introduced the (h, q)-Euler numbers $E_{n,q}^{(h)}$ and polynomials $E_{n,q}^{(h)}(x)$. He gave recurrence identities (h, q)-Euler polynomials and the alternating sums of powers of consecutive (h, q)-integers. In [13], we described the beautiful zeros of the (h, q)-Euler polynomials $E_{n,q}^{(h)}(x)$ using a numerical investigation. Also we investigated distribution and structure of the zeros of the (h, q)-Euler polynomials $E_{n,q}^{(h)}(x)$ by using computer.

Let us define the (h, q)-Euler numbers $E_{n,q}^{(h)}$ and polynomials $E_{n,q}^{(h)}(x)$ as follows:

$$F_q^{(h)}(t) = \frac{2}{q^h e^t + 1} = \sum_{n=0}^{\infty} E_{n,q}^{(h)} \frac{t^n}{n!}, \tag{2}$$

$$F_q^{(h)}(x, t) = \frac{2}{q^h e^t + 1} e^{xt} = \sum_{n=0}^{\infty} E_{n,q}^{(h)}(x) \frac{t^n}{n!}. \tag{3}$$

Observe that if $q \to 1$, then $E_{n,q}^{(h)}(x) = E_n(x)$ and $E_{n,q}^{(h)} = E_n$, where $E_n(x)$ and E_n denote the Euler polynomials and the numbers, respectively (see [2, 5, 8, 16, 17]).

findmeacondoshow.com: Zeros of Bernoulli, Generalized Bernoulli, and Euler Polynomials ( ): Karl Dilcher: Books.Zeros of Bernoulli, Generalized Bernoulli, and Euler Polynomials. Front Cover American Mathematical Society, - Bernoulli polynomials - 94 pages.Title, Zeros of Bernoulli, Generalized Bernoulli, and Euler Polynomials, Issue . Volume of American Mathematical Society: Memoirs of the American.Zeros of Bernoulli Polynomials, Generalized Bernoulli and Euler Polynomials. Front Cover. Karl H. Dilcher. American Mathematical Society, - 94 pages.Zeros of Bernoulli, generalized Bernoulli, and Euler polynomials / Bernoulli polynomials. Euler polynomials. Tags: Add Tag. No Tags, Be the first to tag this.In the year , Rai [5] extended the Bernoulli and Euler polynomials, due to Nath. [3], in two Eai [5] introduced the generalized Bernoulli and Euler polyno- mials by means of the) has zeros at x = 0, y= 0 and x-lh, y= 0. Also, we have.K DilcherAsymptotic behaviour of Bernoulli, Euler and generalized Bernoulli polynomials. J. Approx. K InkeriThe real roots of Bernoulli polynomials. Ann. Univ.K DilcherIrreducibility and zeros of generalized Bernoulli polynomials. C. R. Math . Rep. Acad. Sci. F.T HowardRoots of Euler polynomials. Pacific J. Math., cerned with the Generalized Bernoulli and Euler polynomials. Furthermore, the . index are zero and the non zero values alternate in sign. These numbers.Howard, F. T. Roots of the Euler polynomials. Pacific Commentarii Mathematici, ; Some Formulae for the Product of Two Bernoulli and Euler Polynomials.Uniform Convergence Behavior of the Bernoulli Polynomials The roots of Bernoulli polynomials, Bn(z), when plotted in the complex plane, .. Zeros of Bernoulli, generalized Bernoulli and Euler polynomials, Mem. Amer.Zeros of Bernoulli, generalized Bernoulli, and Euler polynomials. Book.anyone before him, but he did not give a general formula. The Swiss The Bernoulli polynomials have important application in number theory and classical analysis. . with odd index are zero and the non-zero alternate in sign. To illustrate.generalized Fibonacci polynomials and the famous Bernoulli numbers, and ships between the Chebyshev polynomials of the first kind and the famous Euler numbers, . denote the roots of characteristic polynomial ?2 ? P(x)? ? Q(x) of the.Zeros of Bernoulli, generalized Bernoulli and Euler polynomials Maximum modulus of zeros; 3. Zero-free strips; 4. Parabolic zero-free regions; 5. Real zeros of.[1] An Euler-type formula for $\zeta(2k+1)$, J. Number Theory (), no. . [6] Zeros of Bernoulli, generalized Bernoulli and Euler polynomials, Mem. Amer.The question of the location of real zeros of the Bernoulli polynomials. B,(x) has been subject to thorough investigation by several authors. Lense. [7], Lehmer [6] .